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Angle analysis for the evaluation of in-plane anisotropic properties

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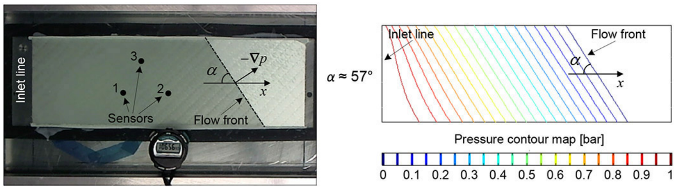
A new approach, based on angle measurements in simple unidirectional tests, reduces the time, samples, and costs required for determination of material parameters such as textile permeability.

In various physical problems, the behavior of a material can be described by constitutive equations in the form of linear relationships between cause and effect vectors.[1] In such equations, the directionally dependent properties of the materials are represented by second-order tensors. An example of such properties is the permeability (K) of a porous medium (i.e., its ability to transmit a fluid because of an applied pressure gradient), which is defined by Darcy's law. Within polymer composite technologies, Darcy's law is commonly used to model the impregnation of a fibrous preform by a liquid resin system.[2] In this context, knowledge of textile permeability is fundamental for simulating the cavity filling and for improving the process design.[2–6]

Different methods can be used to experimentally characterize the permeability of textiles.[4,5,7] For in-plane characterization, unidirectional (1D) injection tests are often performed because these experiments are relatively easy to set up and conduct. In addition, evaluation of the results is simple and a good reproducibility is observed when comparing measurements conducted in different laboratories.[7] The conventional approach for permeability characterization, through 1D tests, is based on injections along different textile directions. In particular, the effective permeability (K_{eff}) is measured along three directions. This allows the determination of the full in-plane permeability tensor and, thus, its principal values (K_1 and K_2).[5,7]

In recent studies,[5,8] we have shown that a complete and accurate in-plane characterization can be achieved by performing injection tests along only one or two textile directions. The new approach is more efficient and cost-effective than the conventional method because it requires fewer experiments and material samples. It is based on concurrent measurements of K_{eff} and the flow front angle (α)—the angle between the flow front and the test direction—as illustrated in Figure 1. We have demonstrated mathematically that α depends on the degree of anisotropy ($\beta = K_2/K_1$) and on the principal permeability orientation (ϑ), according to the following equation:[5]

(A) Test results with unidirectional fibers oriented at 45°



(B) Test results with woven fabric oriented at 45°

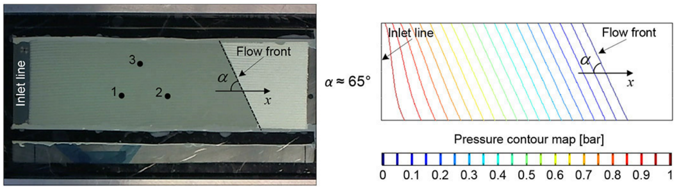


***Figure 1.** Results of unidirectional tests where the textiles were oriented at 45° to the injection direction (x-axis). Pictures on the left show snapshots of the injection experiments on (A) a unidirectional fiber tape and (B) a 2/2 twill woven fabric.[5] Diagrams on the right illustrate the simulated pressure contour lines at the moment of the corresponding snapshot for both textiles. α: Flow front angle. ∇p: Fluid pressure gradient.*

$$\alpha = \tan^{-1}\left(\frac{\sin^2\vartheta + \beta\cos^2\vartheta}{(1-\beta)\sin\vartheta\cos\vartheta}\right) \tag{1}$$

The values of α, as a function of β and ϑ, are illustrated in Figure 2. It is noticeable that when the textile is oriented with a negative ϑ, α is also negative. In these cases, the flow front profile has the opposite slope to that in Figure 1.[5] It should also be noted that the values of 90° and −90° are equivalent in reality, as they indicate that the flow front is perpendicular to the injection direction. These values are obtained

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(see Figure 2) when $\beta = 1$ (i.e., when the permeability is isotropic), or when ϑ is equal to 0° or ±90° (i.e., when the fluid is injected along a principal direction). We have also proved that α represents the slope of the tangent to the permeability ellipse at the intersection with the injection (i.e., the x-axis).^{5} This is illustrated in Figure 3, which also shows that a right angle α arises when the injection direction corresponds to a principal direction (e.g., if ϑ equals 90°).

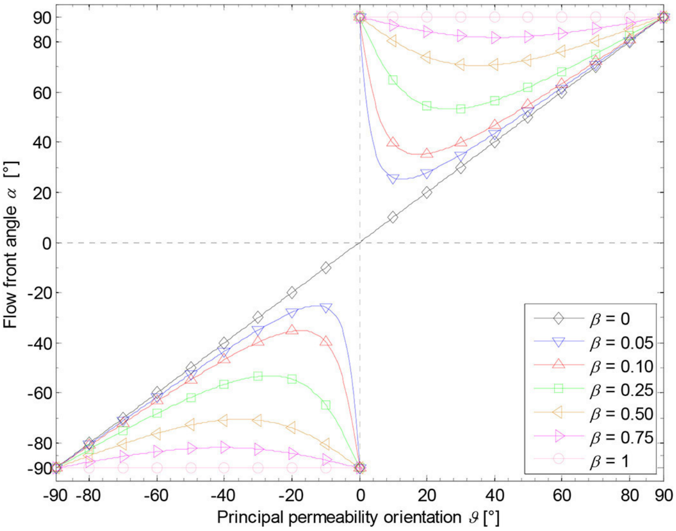

Figure 2. *Flow front angle (α) shown as a function of the principal permeability orientation (ϑ) and the degree of anisotropy (β). When α is equal to ±90°, the flow front is perpendicular to the injection direction. The cases where ϑ is ±90° are also equivalent in reality. The case where β equals 0 is a mathematical limit that is impossible in reality.*

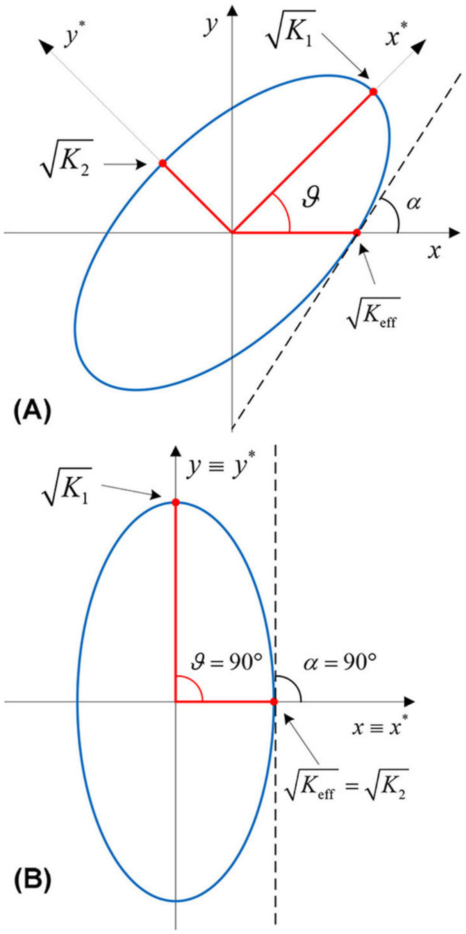

Figure 3. *Permeability ellipses for (A) a generic orientation ϑ and (B) the case when ϑ equals 90°. The x^* and y^* axes represent the principal permeability directions. K_{eff}: Effective permeability. K_1 and K_2: Principal permeability values.*

Measurements of α and K_{eff} along two different textile directions (or along a single non-principal direction, if ϑ is known), allow the unique determination of the permeability ellipse and, thus, the full in-plane characterization of the textile permeability. Based on this concept, we have investigated various characterization strategies.^{5,8} For validation purposes, both virtual tests and real experiments with unidirectional fibers and woven fabrics were conducted.^{5} A good agreement between the results of virtual and real tests for both textile architectures was obtained (see Figure 1). In each case, a stable flow front shape (consistent with the mathematical expression for α in Equation 1) was observed soon after the injection start. It is possible to measure α either by visual inspection of the flow front or by sensing the fluid pressure in the textile, with both measurement techniques returning accurate and comparable results.^{5} Indeed, α is related to the angle between the pressure gradient (∇p), which is perpendicular to the flow front, and the flow velocity (v), which is parallel to the flow direction. Furthermore, isobaric lines have virtually the same slope as the flow front line, except for a small region close to the inlet line (or, more generally, the boundary edges), as shown in Figure 1. Measuring the fluid pressure at three non-aligned sensor points that are sufficiently far from the edges allows ∇p to be reconstructed and α to be calculated from the relative orientation of ∇p with respect to the flow direction. In particular, the flow front angle can be evaluated using the following equation:

$$\alpha = \tan^{-1}\left(\frac{(p_2 - p_1)(y_3 - y_1) - (p_3 - p_1)(y_2 - y_1)}{(p_3 - p_1)(x_2 - x_1) - (p_2 - p_1)(x_3 - x_1)}\right) \qquad (2)$$

where x_i and y_i are the coordinates of the sensor (i) and p_i is its sensed pressure. This equation represents a general formula that is valid

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when the sensors are placed arbitrarily to form the vertices of a triangle (see Figure 1). It can be further simplified, depending on the sensor arrangement.[5]

The use of pressure sensors would allow measurements of α even when the flow front is not visible (e.g., because of an opaque mold) or present (e.g., because of a steady flow through a completely impregnated textile). This new approach is therefore potentially applicable to in-plane characterization of anisotropic permeability in both partially saturated and fully saturated flow conditions.[5] The technique could also be extended for full 3D characterization of the permeability tensor. In such a case, the permeability ellipse (see Figure 3) becomes an ellipsoid and the tangent line is replaced by a tangent plane, which is described by more than one α. This expansion of the approach would require the development of more complex equations, test arrangements, and evaluation methods for the measurable quantities. The approach might also be adapted to the characterization of other material properties that are defined by constitutive equations that belong to the same class of Darcy's law.[1] As for the permeability, such properties are represented by symmetric, positive-definite second-order tensors. For instance—because of the analogy between Darcy's law and Fourier's law of thermal conduction—anisotropic thermal conductivity could be determined using the presented characterization method and substituting temperature sensors for the pressure sensors. In this example of (2D) thermal conduction, α would naturally correspond to the inclination of the isothermal lines, which are perpendicular to the temperature gradient with respect to the direction of heat flux.

To summarize, a new and efficient way to evaluate anisotropic material properties has been developed and verified, focusing on in-plane characterization of textile permeability. The innovative aspect of the approach lies in the use of angle measurements, which have been presented and analyzed in this study. Such angle measurements allow an accurate and cost-effective determination of in-plane permeability through simple 1D tests. Future investigations will address the extension of the methodology to the 3D characterization of permeability or other material properties (e.g., thermal conductivity).

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